Ch-4 Reduction of General equation of Second degree.

* General eq$^n$ of 2nd degree in $x, y, z$

$$ax^2 + by^2 + cz^2 + 2fyz + 2gz + 2hxy + 2ux + 2vy + 2wz + d = 0$$

This equation can be written as

$$F(x,y,z) = f(x,y,z) + 2ux + 2vy + 2wz + d = 0$$

where $f(x,y,z) = ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy$

$$\frac{\partial F}{\partial x} = 2(ax + hy + gz + u)$$

$$\frac{\partial F}{\partial y} = 2(hx + by + fz + v)$$

$$\frac{\partial F}{\partial z} = 2(gx + fy + cz + w)$$

$$\frac{\partial f}{\partial x} = 2(ax + hy + gz)$$

$$\frac{\partial f}{\partial y} = 2(hx + by + fz)$$

$$\frac{\partial f}{\partial z} = 2(gx + fy + cz)$$

* points of Intersection of a line and a conicoid

Find the points of Intersection of the line $\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$ and the surface $f(x,y,z)=0$

$\Rightarrow$ The given line is

$$\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n} = r \text{ (say)} \quad \text{—— (1)}$$

and the given surface is $f(x,y,z) = 0$ —— (2)

Any point on (1) is

$$(rl+\alpha,\ mr+\beta,\ nr+\gamma)$$

It lies on line (2)

$$a(rl+\alpha)^2 + b(\beta+mr)^2 + c(\gamma+nr)^2 + 2f(\beta+mr)(nr+\gamma)$$
$$+ 2g(\gamma+nr)(\alpha+lr) + 2h(\alpha+lr)(\beta+mr) + 2u(\alpha+lr)$$
$$+ 2v(\beta+mr) + 2w(\gamma+nr) + d = 0$$

$$r^2(al^2+bm^2+cn^2+2fmn+2gnl+2hlm)$$
$$+ 2r(l(a\alpha+h\beta+g\gamma+u) + m(h\alpha+b\beta+f\gamma+v)$$
$$+ n(g\alpha+f\beta+c\gamma+w) + f(\alpha,\beta,\gamma) = 0$$

$$r^2 f(l, m, n) + r\left[l\frac{\partial F}{\partial \alpha} + m\frac{\partial F}{\partial \beta} + n\frac{\partial F}{\partial \gamma}\right] + F(\alpha, \beta, \gamma) = 0$$

$\therefore$ It is quadratic in $r$.

* <u>locus of chords Bisected at Given point</u>

find the equation of the plane of the section of the surface $F(x,y,z) = 0$ whose centre is $(\alpha, \beta, \gamma)$

→ Let $(\alpha, \beta, \gamma)$ be the middle point of the chord of the conicoid $F(x,y,z) = 0$

The eqn of chord are

$$\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n} = r, \text{ say} \quad \text{—— (1)}$$

This chord meets the conicoid, where

$$r^2 f(l, m, n) + r\left(l\frac{\partial F}{\partial \alpha} + m\frac{\partial F}{\partial \beta} + n\frac{\partial F}{\partial \gamma}\right) + F(\alpha, \beta, \gamma) = 0 \quad \text{—— (2)}$$

$\because$ $(\alpha, \beta, \gamma)$ is the middle point of the chord

$\therefore$ (2) has two roots equal in magnitude but opposite in sign i.e sum of roots of (2) is zero.

$$l\frac{\partial F}{\partial \alpha} + m\frac{\partial F}{\partial \beta} + n\frac{\partial F}{\partial \gamma} = 0 \quad \text{—— (3)}$$

eliminating $l, m, n$ between ① and ③, the locus of all chords of conicoid which are bisected at $(\alpha, \beta, \gamma)$ is

$$(x-\alpha)\frac{\partial f}{\partial \alpha} + (y-\beta)\frac{\partial f}{\partial \beta} + (z-\gamma)\frac{\partial f}{\partial \gamma} = 0$$

eqn of plane of section of conicoid whose centre is $(\alpha, \beta, \gamma)$.

* <u>Diametral plane</u>

The locus of mid points of all chords parallel to a fixed line $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$ is called diametral plane of the line $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$

* Find the equation of the diametral plane of the conicoid $f(x, y, z) = 0$ which bisects chords parallel to the line $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$

$\Rightarrow$ Let $(\alpha, \beta, \gamma)$ be middle point of the chord.

$\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$ of the conicoid $f(x, y, z) = 0$

$$l \frac{\partial f}{\partial \alpha} + m \frac{\partial f}{\partial \beta} + n \frac{\partial f}{\partial \gamma} = 0$$

$\therefore$ locus of $(\alpha, \beta, \gamma)$ is.

$$= \quad l \frac{\partial f}{\partial x} + m \frac{\partial f}{\partial y} + n \frac{\partial f}{\partial z} = 0$$

$$= \quad l(ax + hy + gz + u) + m(hx + by + fz + v) + n(gx + fy + cz + w) = 0$$

$$= x(al + hm + gn) + y(hl + bm + fn) + z(gl + fm + cn) + (ul + vm + wn) = 0$$